TREATMENT OF ANGULAR DISTRIBUTION OF RADIATION IN THE THEORY OF THE PROPAGATION OF WEAK THERMAL WAVES

V. A. Prokof'ev

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 8, No. 5, pp. 68-79, 1967

This paper establishes criteria for the existence in an infinite medium of weak plane forced waves induced by radiation.

On the basis of the equations of radiation gasdynamics, with an arbitrary (two-parameter) equation of state for the gas, the parameters of wave propagation (attenuation coefficients and a velocity dispersion parameter) have been calculated and analyzed over the entire range of dimensionless numbers characterizing the motion (ratio of specific heats, Boltzmann number, Bouguer number). Thermal self-radiation, absorption of radiation by the gas, and distribution of radiation intensity with direction have been considered. The radiation characteristics assumed were values averaged over frequency. Waves induced by radiation are compared with pressure waves. It is shown that there is a difference between the results obtained and those of an analyisis with radiation intensity averaged over direction.

NOTATION

Here γ is the ratio of specific heats; c_0 is the adiabatic speed of sound; σ is the given cyclic frequency of the forced oscillations; ζ_1^{-1} is the emittance of the gas; Z^{-1} is the Boltzmann number, referred to the

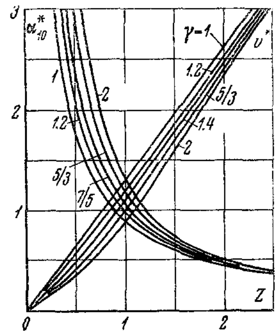

Fig. 1

speed of sound; $(2\pi v)^{-1}$ is the Bouguer number (optical length of the acoustic wave); ω is the volumetric radiation absorption coefficient; $(2\pi\eta)^{-1}$ is the optical length of the wave; $2\pi\alpha_{10}$ is the wave absorption coefficient at the wavelength of an acoustic wave of the same frequency; α_T is the wave absorption coefficient on the mean free path of the radiation; $2\pi\alpha_1$ is the absorption coefficient on a wavelength (true absorption coefficient); r is the velocity dispersion parameter (ratio of the wave phase velocity to the speed of sound).

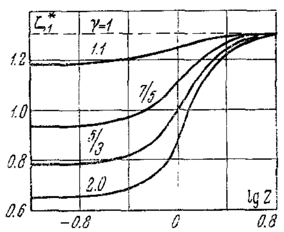

Fig. 2

The propagation of plane forced harmonic perturbations of infinitely small amplitude in an ideal, compressible, resting, infinite fluid is described by the following characteristic equation [1], allowing for influx of heat as a result of thermal radiation:

$$\frac{1}{2q}\ln\frac{1+q}{1-q}=1+\gamma i\zeta_1\frac{1+m^2}{1-m^2},$$

$$q=q_r+iq_i=mv, \qquad m=m_r+im_i=ac_0/\sigma,$$
$$v=\sigma/c_0\omega, \qquad Z=16\sigma' T^3/\rho c_v c_0, \qquad \zeta_1=v/Z. \tag{0.1}$$

Here ρ and T are, respectively, the density and temperature of the undisturbed gas, c_v is the specific heat at constant volume, and σ' is the Stefan-Boltzmann constant. The quantity sought is the complex exponent a of the function $\exp(ax+i\sigma t)$ to which all the gasdynamic

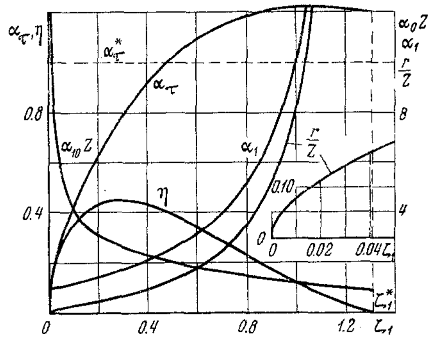

Fig. 3

parameters are proportional, or (which is the same thing) all the dimensionless quantities m or q.

In the derivation we have used a radiative transport equation averaged over the optical frequencies, with accurate allowance for the angular distribution of intensity. On the left side of (0.1) we take one branch of the logarithmic function with argument in the range (0 to π). Equation (0.1) is an even function of m, the signs of the real and imaginary parts of each root being identical: the symmetric plane attenuating waves move out in both directions from the coordinate origin.

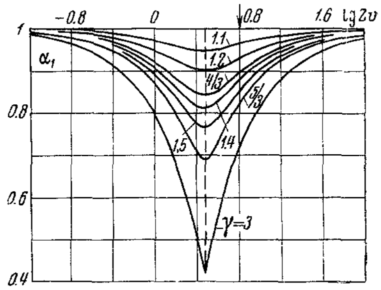

Fig. 4

For any values of Z, v, and γ there exists a pair of roots ($\pm m^{(1)}$) of the characteristic equation, describing the propagation of the pressure waves (see [1]).

Under certain conditions there is also a second pair of roots ($\pm$m). These roots correspond to thermal radiative waves excited in the medium by radiative heat exchange. There are no other cases. We shall examine below the positive real and imaginary parts of the roots, which does not restrict the generality of the conclusions.

The existence of radiation-induced waves which have no analogs in the hydrodynamics of a nonabsorbing gas has been established earlier [2-6]. All the wave parameters are easily expressed in terms of the real part m_r and the imaginary part m_i of the roots

$$\alpha_{10}=m_r, \quad \alpha_\tau=q_r=m_r v, \quad \alpha_1=m_r/m_i,$$
$$\eta=m_i v, \quad r=m_i^{-1}, \quad \alpha_a^1=\sigma c_0^{-1}m_r. \tag{0.2}$$

Here α_a^1 are the wave attenuation coefficients per unit length.

§1. CONDITIONS FOR THE EXISTENCE OF WAVES INDUCED BY RADIATION

It follows from (0.1) that for each value of γ and Z, the real and imaginary parts of the second root are monotonically decreasing functions of v. As v tends toward some value v^*, the imaginary part of the root tends to zero (the wave velocity then tends to infinity). The real

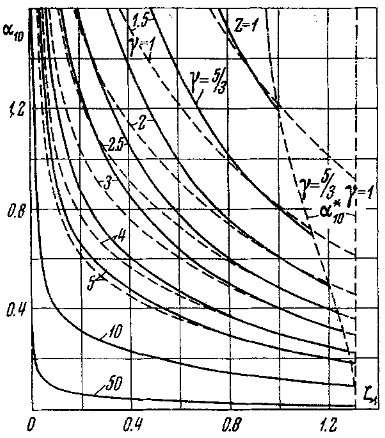

Fig. 5

part of the root differs from zero in the entire region $v < v^*$. For all values $v \geq v^*$ there is no second root. The limiting values of the parameters (denoted by asterisks) are determined by Eq. (0.1), if m_i tends to zero there:

$$\alpha_\tau{}^* = \operatorname{cth} \alpha_\tau{}^*, \quad \pi(\gamma v^{*2} + \alpha_\tau{}^{*2}) = 2\gamma\zeta_1\alpha_\tau{}^*(v^{*2} + \alpha_\tau{}^{*2}). \quad (1.1)$$

The root $\alpha_T^* = 1.199678$ of the first equation [7] also corresponds to the vanishing of the radiative flux. The second equation imposes restrictions on ζ_1 (or Z), v, and γ. Hence, for each value of γ and Z, a unique positive value of v^* or ζ_1^* (Figs. 1 and 2) is determined.

In the case of a piezotropic medium ($\gamma = 1$), not only α_T^*, but also ζ_1^* will be universal constants. From (1.1) we have

$$v^* = {}^1/_2\pi Z / \alpha_\tau{}^* = 1.309348\,Z, \quad \zeta_1{}^* = {}^1/_2\pi / \alpha_\tau{}^* = 1.309348,$$
$$\zeta^* = \zeta_1{}^{*-1} = 0.7637383, \quad \alpha_{10}{}^* = 0.916240Z^{-1}. \quad (1.2)$$

For any values of γ and Z, the number ζ_1^* lies in the interval

$$\pi(2\gamma\alpha_\tau{}^*)^{-1} \leqslant \zeta_1{}^* \leqslant \pi(2\alpha_\tau{}^*)^{-1}. \quad (1.3)$$

For $Z \ll 1$ ($s = {}^1/_2\pi Z\alpha_T{}^{*-2} = 1.091417Z$),

$$\alpha_{10}{}^* = \gamma s^{-1}[1 - (\gamma - 1)\gamma^{-2}s^2 + (\gamma - 1)(2 - \gamma)\gamma^{-4}s^4 + \cdots],$$
$$v^* = \alpha_\tau{}^*\gamma s^{-1}[1 + (\gamma - 1)\gamma^{-2}s^2 + (\gamma - 1)(2\gamma - 3)\gamma^{-4}s^4 + \cdots]. \quad (1.4)$$

For $Z \gg 1$, the dependence of the parameters on γ vanishes,

$$\alpha_{10}{}^* = s^{-1}[1 + (\gamma - 1)\gamma^{-1}s^{-2} + (\gamma - 1)(2\gamma - 3)\gamma^{-2}s^{-4} + \cdots],$$
$$v^* = \alpha_\tau{}^* s[1 - (\gamma - 1)\gamma^{-1}s^{-2} + (\gamma - 1)(2 - \gamma)\gamma^{-2}s^{-4} + \cdots]. \quad (1.5)$$

If γ is close to unity ($\gamma - 1 = \delta \ll 1$) we have

$$\alpha_{10}{}^* = \frac{1}{s}\left[1 + \frac{\delta}{1 + s^2} + \frac{s^2(1 - s^2)}{(1 + s^2)^3}\delta^2 + \right.$$
$$\left. + \frac{s^2(s^6 - 2s^4 + 4s^2 - 1)}{(1 + s^2)^5}\delta^3 + \cdots\right], \quad (1.6)$$

$$v^* = \alpha_\tau{}^* s\left[1 - \frac{\delta}{1 + s^2} + \frac{1 + s^4}{(1 + s^2)^3}\delta^2 - \right.$$
$$\left. - \frac{s^6 + 5s^4 - s^2 + 1}{(1 + s^2)^5}\delta^3 + \cdots\right]. \quad (1.7)$$

As Z increases, the quantity v^* and its derivative increase monotonically from the values in (1.4), which are proportional to Z and inversely proportional to γ, to the values in (1.5), which are inversely proportional to Z, and asymptotically approach the values for a piezotropic medium. For any value of Z, v^* is the smaller, the larger the value of γ. The value of ζ_1^* increases monotonically with increase of Z from $\pi(2\gamma\alpha_T{}^*)^{-1}$ to its value $\pi(2\alpha_T{}^*)^{-1}$ for a piezotropic medium. The curve $\zeta_1^*(Z)$ has a single point of inflection. For any value of Z, the value of ζ_1^* is the larger, the smaller γ.

Figure 1 also gives the function $Z^*(v)$ or $Z^*(l_{T0})$. For given values of γ and l_{T0}, thermal waves are formed only for $Z > Z^*$. For each given optical length l_{T0} of an acoustic wave, there is a limiting value of Z below which thermal waves are not excited: beyond the acoustic wave limit, the amount of radiation energy released is insufficient to induce new waves. For given values of γ and Z, further waves are excited only at frequencies corresponding to acoustic waves with optical lengths not less than $l_{T0}{}^* = 2\pi v^{*-1}$.

With increase of v from zero to v^*, the radiative flux increases from 0 to a maximum, and then falls to 0 as $v \to v^*$. The limit value of the absorption coefficient α_{10}^* per unit length of acoustic adiabatic wave decreases monotonically with increasing Z (Fig. 1); its value is the greater, the greater the value of γ for the same value of Z. For large Z it coincides with its value in a piezotropic medium. The absorption coefficient per radiation mean free path is equal in the limit to the constant value α_T^*, for all values of γ and Z. The velocity and length of the wave become infinite in the limit, and harmonic perturbations of the radiation field in the coordinate plane generate exponentially attenuating (in space) oscillations of the entire medium as a whole. Traveling waves are not formed.

Thermal radiation waves exist for $\zeta_1 > \zeta_1^*$. This condition, because of Eq. (1.10) in [1], may be rewritten as

$$\frac{\varepsilon_2}{\varepsilon_T} > \frac{\varepsilon_2{}^*}{\varepsilon_T{}^*} = \frac{1}{\alpha_\tau{}^*}\frac{\gamma v^{*2} + \alpha_\tau{}^{*2}}{\gamma(v^{*2} + \alpha_\tau{}^{*2})}. \quad (1.8)$$

Waves can arise only under conditions in the medium and at frequencies of the forced oscillations such that when the ratio of the radiation energy emitted per unit mass of gas during an oscillation period

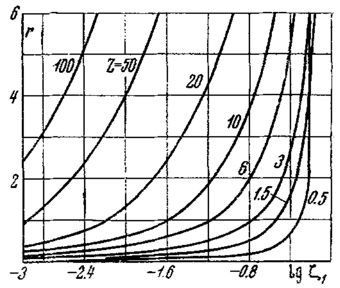

Fig. 6

to the mass density of the internal thermal energy of the gas becomes no less than some limit quantity on the order of unity.

The behavior of the wave parameters in the neighborhood of the limit point $\alpha_T = \alpha_T^*$, $\eta = 0$ is determined by the relations

$$\alpha_\tau = \alpha_\tau{}^* + A_1(v - v^*), \qquad \eta = A_2(v - v^*),$$
$$A_1 = -\gamma R^{-1}\alpha_\tau{}^*\zeta_1{}^*\varphi^*(\alpha_\tau{}^* - 1)^2\{(v^2 + \alpha_\tau{}^{*2})\chi^{*2} +$$
$$+ 2v^{*2}[2\gamma\chi^{*2} - \psi^{*2}(\chi^* + \gamma\psi^*)]\},$$
$$R = \gamma^2 Z\zeta_1{}^{*2}(\alpha_\tau{}^{*2} - 1)^2[(v^{*2} + 5\alpha_\tau{}^{*2})\chi^{*2} -$$
$$- 2\alpha_\tau{}^{*2}\psi^*(\chi^* + \psi^*)] + \alpha_\tau{}^{*4}(\gamma v^{*2} + \alpha_\tau{}^{*2})\chi^{*3},$$
$$\psi^* = \pi\chi^*(2\gamma\zeta_1{}^*\alpha_\tau{}^*)^{-1},$$
$$A_2 = \chi^*\alpha_\tau{}^{*2}A_1[\gamma\zeta_1{}^*\varphi^*(\alpha_\tau{}^{*2} - 1)]^{-1},$$
$$\varphi^* = \psi^* + 2(\gamma - 1)\alpha_\tau{}^{*2}v^{*2}, \quad \chi^* = (\gamma v^{*2} + \alpha_\tau{}^{*2})^2. \quad (1.9)$$

The quantities A_1 and A_2 are negative, decrease monotonically in absolute magnitude with increasing Z (and v), and increase with increase of γ ($1 \leq \gamma \leq 2$) i.e.,

$$A_1 \approx -0.12615\gamma Z^{-1}, \quad A_2 \approx -0.31570\gamma Z^{-1} \quad (Z \ll 1),$$
$$\left.\begin{aligned} A_1 &\approx -0.12615\,[\gamma + 4(\gamma^2-1)]\,(\gamma Z)^{-1}, \\ A_2 &\approx -0.31570\,[\gamma + 4(\gamma^2-1)]\,(\gamma Z)^{-1}, \end{aligned}\right. \quad (Z \gg 1). \tag{1.10}$$

§2. PIEZOTROPIC MEDIUM

The laws of wave propagation in a gas, even for $\gamma \neq 1$, beginning at some values of ζ_1, coincide in first approximation with the laws for a piezotropic medium, if Z is large enough, and they are given by similar laws, if Z is small. Equation (0.1) for $\gamma = 1$ decomposes into the two parts

$$m^2 + 1 = 0, \quad {}^1\!/\!{}_2 q^{-1} \ln [(1+q)/(1-q)] = 1 + i\zeta_1. \tag{2.1}$$

The first equation describes motion at the speed of sound for nonattenuating pressure waves. The second equation gives the laws of motion for thermal radiation waves. It is equivalent to the system

$$\mu = \alpha_\tau - \zeta_1\eta, \qquad \nu = \eta + \zeta_1\alpha_\tau,$$
$$\mu \equiv \frac{1}{4}\ln\frac{(1+\alpha_\tau)^2+\eta^2}{(1-\alpha_\tau)^2+\eta^2},$$
$$\nu \equiv \begin{cases} {}^1\!/\!{}_2 \operatorname{arc\,tg}\,[2\eta(1-\alpha_\tau^2-\eta^2)^{-1}], & \alpha_\tau^2+\eta^2<1 \\ {}^1\!/\!{}_2 \{\pi - \operatorname{arc\,tg}\,[2\eta(\alpha_\tau^2+\eta^2-1)^{-1}]\}, & \alpha_\tau^2+\eta^2>. \end{cases} \tag{2.2}$$

The desired root depends only on ζ_1. For $\alpha_T^2 + \eta^2 = 1$ we obtain $\alpha_T = 0.89666540$, $\eta = 0.44270888$, $\zeta_1 = 0.38218189$. It is clear from (2.2) that $\alpha_1 \geq 1$ (the equality with $\alpha_T = \eta = 0$), the shape of the harmonic oscillations in the wave is not maintained, and disturbances attenuating almost exponentially are induced. For small values of ζ_1, the solution is given by the expansions

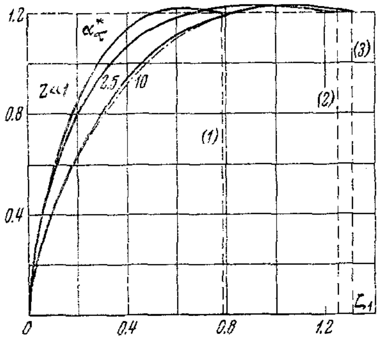

Fig. 7

$$\alpha_\tau = \sqrt{1.5\zeta_1}\,(1 + a_1\zeta_1 - a_2\zeta_1^2 - a_3\zeta_1^3 - a_4\zeta_1^4 + \cdots),$$
$$\eta = \sqrt{1.5\zeta_1}\,(1 - a_1\zeta_1 - a_2\zeta_1^2 + a_3\zeta_1^3 - a_4\zeta_1^4 + \cdots),$$
$$\alpha_1 = 1 + 2a_1\zeta_1 + 2a_1^2\zeta_1^2 + 2(a_1^3 + a_1a_2 - a_3)\zeta_1^3 + $$
$$+ 2a_1(a_1^3 - 2a_1a_2 - 2a_3)\zeta_1^4 + \cdots,$$
$$r = \sqrt{2z_1}\,[1 + a_1\zeta_1 + (a_1^2 + a_2)\zeta_1^2 + (a_1^3 + 2a_1a_2 - a_3)\zeta_1^3 +$$
$$+ (a_1^4 + 3a_1^2a_2 - 2a_1a_3 + a_2^2 + a_4)\zeta_1^4 + \cdots],$$
$$z_1 = \frac{1}{3}Zv, \quad a_1 = \frac{9}{10}, \quad a_2 = \frac{1269}{1400},$$
$$a_3 = \frac{12\,699}{14\,000}, \quad a_4 = \frac{450\,672\,741}{21\,560\,000}. \tag{2.3}$$

In first approximation α_{10} and r depend only on z_1. In the vicinity of the limit value ζ_1^*,

$$\alpha_\tau = \alpha_\tau^* - 0.12615\,(\zeta_1 - \zeta_1^*),$$
$$\eta = -0.31570\,(\zeta_1 - \zeta_1^*). \tag{2.4}$$

The parameters α_T, $Z\,\alpha_{10}$, α_1, rZ^{-1}, η (Fig. 3) are determined only by the quantity ζ_1. With increase of ζ_1, the coefficient α_T increases from 0 to a maximum value (equal to ~1.22 for $\zeta_{1\max} \approx$ ≈ 1.05 and then falls to the limiting value α_T^*. The optical wave number η also has a unique maximum (equal to ~0.46 for $\zeta_1 \approx 0.29$). The product $Z\alpha_{10}$ decreases monotonically from ∞ to a limiting value. For a fixed value of ζ_1, the absorption coefficient $\alpha_{10} \sim Z^{-1}$, and for fixed Z, it is the smaller, the greater v. The absorption coefficient α_1 increases monotonically from 1 to ∞. The quantity rZ^{-1} increases monotonically from 0 to ∞ with increasing ζ_1 from 0 to ζ_1^*. For a fixed value of ζ_1, the ratio of velocities $r \sim Z$. At the coordinate origin $dr/dv = \infty$.

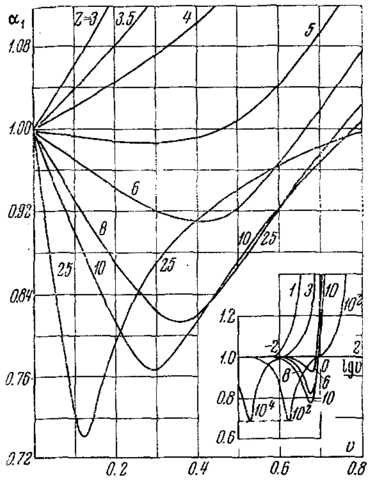

Fig. 8

From Fig. 3 we can also obtain information regarding the dependence of the parameters on ν. The curve $\alpha_1(v)$ goes to values that are the lower, the larger Z; curves corresponding to various values of Z do not intersect. The coefficient $\alpha_{10} \sim Z^{-1/2}$ for $v \ll 1$. The two curves of $\alpha_{10}(v)$ corresponding to Z_1 and Z_2 either do not intersect, or intersect at a single point. Intersection occurs if v simultaneously satisfies the inequalities

$$\zeta_{1\max} Z_1 < v < \zeta_1^* Z_1, \quad \zeta_1^{**} Z_2 \leqslant v < \zeta_{1\max} Z_2, \quad Z_1 < Z_2. \tag{2.5}$$

(Here ζ_1^{**} is the smallest root of the equation $\alpha_T^*(\zeta_1) = 1$.)

The curve $\alpha_T(v)$ has a maximum. The ascending branch of the curve is the higher, the smaller Z. The ascending branch of the curve, corresponding to the larger Z can intersect at one point with the descending branch, corresponding to small Z, if inequalities (2.5) can be satisfied simultaneously. Replacing Z_1 in (2.5) by Z_2 ($Z_1 > Z_2$), we

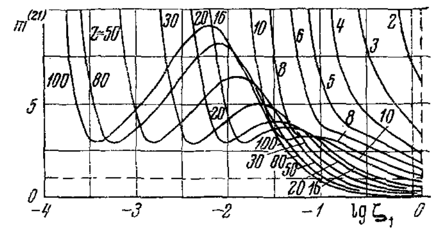

Fig. 9

obtain the conditions for the intersection of the descending branch of the curve corresponding to the smaller Z, with the ascending branch corresponding to the larger Z.

The curve r(v) increases monotonically from 0 to ∞, and at the coordinate origin the derivatives $dr/dv = \infty$. For small values of v, the curve goes to values that are the greater, the greater the value of Z. A curve corresponding to a fixed value of Z intersects once with each curve corresponding to another value of Z; the point of intersection lies increasingly to the right and higher, the larger the value of Z corresponding to the second curve. At any point in the positive quadrant of the plane (r, v), two curves corresponding to different values of Z intersect.

With increasing v, the optical wave number η increases the faster, the smaller the value of Z. The quantity $\eta \sim Z^{-1/2}$ for small v. A maximum occurs in the curve, after which $\eta(v)$ decreases from 0 to the limiting point. The curves for various Z each intersect once (not counting the origin of the coordinates), the ascending branch of the curve corresponding to the larger value of Z, intersecting with the

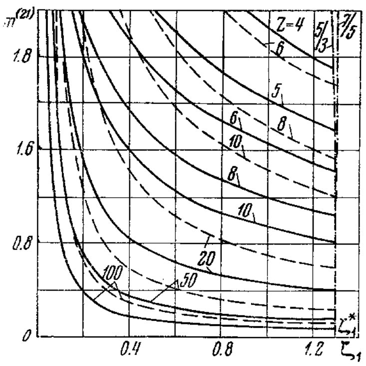

Fig. 10

decreasing branch which corresponds to the smaller Z.

Induced waves are attenuated, whereas pressure waves are nonattenuating; at all frequencies pressure waves predominate.

§3. SMALL VALUES OF Z

For $Z \ll 1$, thermal radiation waves exist only for $v \ll 1$, with $|m| \gg 1$, $|q| \leq 0(1)$ throughout. In first approximation, Eq. (0.1) takes the form

$$\frac{1}{2q} \ln \frac{1+q}{1-q} = 1 + \gamma i \zeta_1, \qquad (3.1)$$

which differs from (2.2) in that ζ_1 is replaced by $\gamma\zeta_1$. Subsequent approximations can be determined by putting (m_0 is a root of Eq. (3.1))

$$m = m_0 (1 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \cdots). \qquad (3.2)$$

The second approximation $m_0\varepsilon_1$ and the third approximation $m_0\varepsilon_2$ for $\zeta_1 = 0(1)$ are determined by the relations ($\alpha_{\tau 0} = m_{r0}\nu$, $\eta_0 = m_{i0}v$)

$$\operatorname{Re} \varepsilon_1 = D[2A\alpha_{\tau 0}\eta_0 - B(\alpha_{\tau 0}^2 - \eta_0^2)],$$

$$\operatorname{Im} \varepsilon_1 = D[A(\alpha_{\tau 0}^2 - \eta_0^2) - 2B\alpha_{\tau 0}\eta_0],$$

$$A = 1 - C(1 - \alpha_{\tau 0}^2 - \eta_0^2),$$

$$C = [(1 - \alpha_{\tau 0}^2 + \eta_0^2)^2 + 4\alpha_{\tau 0}^2\eta_0^2]^{-1}, \quad B = \gamma\zeta_1 + 2\alpha_{\tau 0}\eta_0 C,$$

$$D = \gamma(\gamma - 1) Zv (\alpha_{\tau 0}^2 + \eta_0^2)^{-2} (A^2 + B^2)^{-1}, \qquad (3.3)$$

$$m_0^4 [1 + \gamma i\zeta_1 + (m_0^2 v^2 - 1)^{-1}] \varepsilon_2 = m_0^4 \varepsilon_1^2 [1 + \gamma i \zeta_1 + $$
$$+ (2m_0^2v^2 - 1)(m_0^2v^2 - 1)^{-2}] - (\gamma - 1)\gamma i\zeta_1 (m_0^2\varepsilon_1 + \gamma). \qquad (3.4)$$

For small values of ζ_1 (with $v \ll 1$, $Zv \ll 1$ simultaneously)

$$m_0 v = (1 + i)(3/2\,\gamma\zeta_1)^{1/2}, \quad \varepsilon_1 = -ic_1\zeta_1,$$

$$\varepsilon_2 = -c_2\zeta_1^2, \quad \varepsilon_3 = -ic_3\zeta_1^3, \qquad (3.5)$$

$$c_1 = 9/10\,\gamma(1 - Z^2/Z_0^2), \qquad Z_0^2 = 5.4\gamma^2/(\gamma - 1),$$

$$c_2 = 1269/1400\,\gamma^2 - 3/20\,(\gamma - 1)Z^2 + 1/72\,(\gamma - 1)(5 - \gamma)\gamma^{-2}Z^4,$$

$$c_3 = 2c_1^3 + 1/6\,c_1^2 (Z^2 - 81\gamma) - 6/35\,\gamma c_1 (7Z^2 - 90\gamma) -$$
$$- c_2[c_1 + 1/3\,(2\gamma - 3)Z^2] + 9/14\,\gamma^2(Z^2 - 7\gamma),$$

$$\alpha_{10} = Z^{-1}(3/2\,\gamma\zeta_1^{-1})^{1/2}(1 + c_1\zeta_1 - c_2\zeta_1^2 + c_3\zeta_1^3 + \cdots),$$

$$m_i = Z^{-1}(3/2\,\gamma\zeta_1^{-1})^{1/2}(1 - c_1\zeta_1 - c_2\zeta_1^2 - c_3\zeta_1^3 + \cdots),$$

$$\alpha_1 = 1 + 2c_1\zeta_1 - 2c_1^2\zeta_1^2 + 2(c_1^3 + c_1c_2 + c_3)\zeta_1^3 + \cdots),$$

$$r = Z(2/3\,\gamma^{-1}\zeta_1)^{1/2}[1 + c_1\zeta_1 + (c_1^2 + c_2)\zeta_1^2 +$$
$$+ (c_1^3 + 2c_1c_2 + c_3)\zeta_1^3 + \cdots]. \qquad (3.6)$$

In the neighborhood of $\zeta_1 = \zeta_1^*$

$$\alpha_\tau = [1 + (1 + \lambda^2)^{-1}(1 - v/v^*)]\,\alpha_\tau^*,$$

$$\eta = \frac{\lambda\alpha_\tau^*}{1 + \lambda^2}\left(1 - \frac{v}{v^*}\right), \qquad \lambda = \frac{\alpha_\tau^{*2}}{\gamma\zeta_1^*(\alpha_\tau^{*2} - 1)}. \qquad (3.7)$$

Thus, if $Z \ll 1$, then $\alpha_{10} \gg 1$ for any value of $v < v^*$ and it decreases with increasing v; for small values of v, the coefficient α_{10} is inversely proportional to the square root of the frequency. Thermal radiation waves for $Z \ll 1$ are attenuated much more rapidly than pressure waves of the same frequency. The coefficient $\alpha_1 \gg 1$ also increases with an increase or decrease in Z, and the shape of the wave becomes distorted. More than likely, these are initial disturbances attenuating exponentially, according to a somewhat changed Bouguer law. The waves are not formed in reality. The velocity of propagation is small for $v \ll 1$, and is proportional to the square root of the frequency; with increasing v the velocity and the wavelength increase monotonically from 0 to ∞. The coefficients α_τ and η increase from 0 to a maximum, following which $\alpha_\tau \to \alpha_\tau^*$ and $\eta \to 0$.

§4. LARGE VALUES OF Z

For large values of Z a solution describing thermal radiation waves exists over a wide range of values of v.

1) For $v \ll 1$, $Zv \ll 1$, expansions (3.6) are valid. The thermal radiation waves are attenuated much more strongly, they are propagated much more slowly than the pressure waves, and they are much shorter than the latter. With increasing v, the coefficients α_1 and α_{10} decrease, and the quantities r and α_τ increase. The coefficients α_{10} and α_τ are the greater, the greater the value of γ and the smaller the value of Z. The velocity and the wavelength are the larger, the smaller the value of γ and the larger the value of Z. The coefficient α_1 decreases with increasing γ and Z.

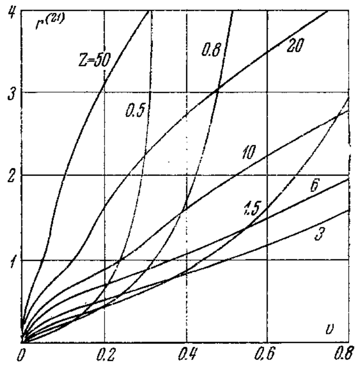

Fig. 11

2) For $v \leq 1$, $Zv = 0(1)$ the desired root of the characteristic equation is given by expansions (4.2) of reference [1] where

$$m_{r0} = \frac{u_1}{\sqrt{2z_1}}, \quad m_{i0} = \frac{u_1}{\sqrt{2z_1}} \quad (1 \leqslant \gamma < 2), \qquad (4.1)$$

$$u_1 = [1/2\,\gamma(a_5 + a_2 - z_1)]^{1/2}, \qquad u_2 = [1/2\,\gamma(a_5 - a_2 + z_1)]^{1/2},$$

$$a_5 = [1 + a_1 + z_1^2 - 2(a_2z_1 - a_3)]^{1/2}, \qquad (4.2)$$

and the quantities a_1, a_2, and a_3 are given in [1]. In first approximation

$$\alpha_{10} = u_1/\sqrt{2z_1}, \quad \alpha_\tau = u_1\sqrt{3/2\,\zeta_1}, \quad \alpha_1 = u_1/u_2,$$

$$r = \sqrt{2z_1}/u_2, \quad \eta = u_2\sqrt{3/2\,\zeta_1}. \qquad (4.3)$$

These expressions also yield formulas in first approximation when $Zv \ll 1$ and $v \ll 1$, $Zv \gg 1$, i.e., they are suitable in first approximation, when $v \ll 1$, $\zeta_1 \ll 1$. Both pairs of roots are of order 1 and are given in first approximation by the values of Zv and γ. The coefficient α_{10} decreases monotonically with increasing Zv. The coefficient α_T is a small quantity, increasing monotonically with Zv. For a fixed value of Zv, the coefficient $\alpha_T \sim Z^{-1}$. The coefficient α_1 (Fig. 4) has a minimum

$$z_{1\,\min} = 1\,,$$
$$\alpha_{1\,\min} = (1 + \sqrt{2-\gamma}) / (1 + \sqrt{\gamma})\,. \tag{4.4}$$

The minimum is the smaller, the larger γ (for $\gamma = 2$, $^5/_3$, $^3/_2$, $^7/_5$, $^4/_3$, $^6/_5$, $^{11}/_{10}$ the values are, respectively, $\alpha_{10\min} = 0.4142136$, 0.6885003, 0.7673269, 0.8128361, 0.8430390, 0.9041691, 0.9511225). The wave velocity increases monotonically with increasing Zv, and in this region there is a transition from subsonic to supersonic velocity. The quantity η is small, increases monotonically with increase of v (for given Z), and decreases monotonically with increase of Z (for given v). In the region Zv = 0 (1) the curves of $\alpha_{10}(v)$ (and also of $\alpha_{10}(\zeta_1)$) for the same values of Z but different values of γ intersect, and after intersection the curve corresponding to the smaller value of γ lies higher.

The ratio $m^{(21)}$ of attenuation coefficients for thermal radiation waves and pressure waves at a fixed length, and the ratio $\alpha_1^{(21)}$ of the true attenuation coefficients in this region reach minimum values greater than unity. For any value of Z, and for small v, the pressure waves predominate. The ratio of velocities and wavelengths $r^{(21)}$ increases monotonically with increase of Zv, and reaches unity for $z_1 = \gamma/2$.

If we consider $\gamma \geq 2$, then for $\gamma = 2$ we have

$z_1 < 1$

$$\alpha_{10} = \sqrt{{}^1/_2 (y_0 - 1)}, \qquad m_{i0} = \sqrt{{}^1/_2 (y_0 + 1)},$$
$$\alpha_1 = [(y_0 + 1) / (y_0 - 1)]^{1/2}, \quad y_0 = z_1^{-1} [2(1 + \sqrt{1 - z_1^2})]^{1/2}\,; \tag{4.5}$$

$z_1 = 1 \mp 0$

$$\alpha_{10} = \sqrt{{}^1/_2 \alpha_1}, \qquad r = \sqrt{2\alpha_1},$$
$$\alpha_1 = \sqrt{2} - 1, \qquad d\alpha_1 / dz_1 = \mp\infty\,; \tag{4.6}$$

$z_1 > 1$

$$\alpha_{10} = \sqrt{1 - y_0'} \qquad \alpha_1 = \left(\frac{1 - y_0'}{1 + y_0'}\right)^{1/2},$$
$$m_{i0} = \sqrt{1 + y_0'},$$
$$y_0' = \left(\frac{z_1 - \sqrt{z_1^2 - 1}}{2z_1}\right)^{1/2}. \tag{4.7}$$

If $\gamma > 2$, then in place of a_5 in (4.1) we must put

$$a_5' = [1 + a_1 + z_1^2 - 2(a_2 z_1 + a_3)]^{1/2}\,. \tag{4.8}$$

At the point $z_1 = 1$ for $1 < \gamma < 2$, the pressure wave absorption coefficients $\alpha_{10}^{(1)}$ and $\alpha_1^{(1)}$ [1] have maxima, while for thermal radiation waves α_1 and the ratio $\alpha^{(21)}$ have minima. The parameters of both waves at this point are

$$m_r^{(1)} = {}^1/_2 [\sqrt{\gamma}(2 - \sqrt{2-\gamma} - \sqrt{\gamma})]^{1/2},$$
$$m_i^{(1)} = {}^1/_2 [\sqrt{\gamma}(2 + \sqrt{2+\gamma} + \sqrt{\gamma})]^{1/2},$$
$$\alpha_1^{(1)} = (1 - \sqrt{2-\gamma})(1 + \sqrt{\gamma})^{-1} < 1\,,$$
$$m_r = {}^1/_2 [\sqrt{\gamma}(2 + \sqrt{2-\gamma} - \sqrt{\gamma})]^{1/2},$$
$$m_i = {}^1/_2 [\sqrt{\gamma}(2 - \sqrt{2-\gamma} + \sqrt{\gamma})]^{1/2}$$
$$\alpha_1 = (1 + \sqrt{2-\gamma})(1 + \sqrt{\gamma})^{-1} < 1\,,$$
$$m^{(21)} = \frac{2 + \sqrt{2-\gamma} - \sqrt{\gamma}}{\sqrt{2}(\sqrt{\gamma} - 1)}, \qquad \alpha^{(21)} = \frac{1 + \sqrt{2-\gamma}}{1 - \sqrt{2-\gamma}},$$
$$r^{(21)} = \left(\frac{2 + \sqrt{2-\gamma} + \sqrt{\gamma}}{2 - \sqrt{2-\gamma} + \sqrt{\gamma}}\right)^{1/2}. \tag{4.9}$$

At the same point, for $\gamma \geq 2$,

$$m_r^{(1)} = {}^1/_2 [(\sqrt{2} - 1)(\gamma - \sqrt{\gamma(\gamma-2)})]^{1/2}$$
$$m_i^{(1)} = {}^1/_2 [(\sqrt{2} + 1)(\gamma - \sqrt{\gamma(\gamma-2)})]^{1/2},$$
$$\alpha_1^{(1)} = \alpha_1 = \sqrt{2} - 1$$
$$m_r = {}^1/_2 [(\sqrt{2} - 1)(\gamma + \sqrt{\gamma(\gamma-2)})]^{1/2}$$
$$m_i = {}^1/_2 [(\sqrt{2} + 1)(\gamma + \sqrt{\gamma(\gamma-2)})]^{1/2}$$
$$m^{(21)} = {}^1/_2 \sqrt{2}(\sqrt{\gamma} + \sqrt{\gamma-2}) \geqslant 1,$$
$$r^{(21)} = {}^1/_2 \sqrt{2}(\sqrt{\gamma} - \sqrt{\gamma-2}) \leqslant 1, \quad \alpha^{(21)} = 1 \tag{4.10}$$

The coefficients α_1 of both kinds of waves in this region, for any γ, fall between 0 and 1, and for $z_1 = 1$ the coefficients of the different kinds of waves draw together with increasing γ, and for $\gamma = 2$ they merge, as do the coefficients α_{10}. With further increase of γ for $z_1 = 1$, an angle point is formed if the pressure waves are understood to be those waves which are continuously converting to sound waves at the edges of the region (Z, v).

3) In the case $\zeta_1 \ll 1$, $Zv \gg 1$ for any values of Z and v, we have

$$\alpha_{10} = {}^1/_2 z_1^{-1} \sqrt{2z_1} [1 - f_1 + f_2 + O(k)],$$
$$\eta = \sqrt{{}^3/_2 \zeta_1} [1 + f_1 + f_2 + O(k)]\,,$$
$$\alpha_1 = 1 = 2f_1 + 2f_1^2 + O(k),$$
$$r = \sqrt{2z_1} [1 - f_1 - f_2 + f_1^2 + O(k)]\,,$$
$$f_1 = (\gamma - 1 - {}^3/_5 \gamma v^2)(2\gamma z_1)^{-1},$$
$$f_2 = [(\gamma - 1)\,,$$
$$(7 - 3\gamma + 6\gamma v^2) - {}^{141}/_{175} \gamma^2 v^4] (8\gamma^2 z_1^2)^{-1}\,,$$
$$k = \zeta_1^m z_1^{-n} \; (n + m = 3;\; n, m = 0, 1, 2, 3)\,. \tag{4.11}$$

In this region α_{10}, α_T, α_1 and r are the larger, the smaller γ.

The thermal radiation waves in this range of Z and v are attenuated weakly over the length of an acoustic wave and over the radiation free path, with $\alpha_{10} \sim v^{-1/2}$, $\alpha_T \sim v^{1/2}$, $\alpha_a^1 \sim v^{1/2}$. The correction to the first approximation of the absorption coefficient is negative for $v < \bar{v}^* \approx [5(\gamma - 1)/(3\gamma)]^{1/2}$, and positive for $v > \bar{v}^*$ (if $\gamma = 2$, $^5/_3$, $^3/_2$, $^7/_5$, $^4/_3$, $^6/_5$, $^{11}/_{10}$, then, correspondingly, $\bar{v}^* = 0.91287$, 0.81650, 0.69007, 0.74536, 0.64550, 0.52705, 0.038925). The coefficient α_1 reaches the value 1 for $v = \bar{v}^*$, after which it continues to increase monotonically along with v. Since α_1 is close to unity, the shape of the wave is distorted. In this region the quantity $\beta \gg 1$ [1] for any value of v, for which reason we have, in first approximation,

$$m^{(21)} = \frac{\sqrt{6\zeta_1}}{\gamma - 1} K', \qquad \alpha^{(21)} = \frac{2\sqrt{\gamma}}{\gamma - 1} ZK',$$
$$r^{(21)} = \sqrt{2\gamma z_1}\,. \tag{4.12}$$

For $v \ll 1$, $Zv \gg 1$, and for $v \gg 1$, $\zeta_1 \ll 1$, we obtain, respectively,

$$m^{(21)} = \left(\frac{2\gamma z_1}{\gamma - 1}\right)^{1/2}, \qquad \alpha^{(21)} = \frac{2\gamma z_1}{\gamma - 1}\,. \tag{4.13}$$

$$m^{(21)} = \frac{\sqrt{6\gamma\zeta_1}}{(\gamma - 1)\gamma v}, \qquad \alpha^{(21)} = \frac{2\sqrt{\gamma}}{(\gamma - 1)\zeta_1}\,. \tag{4.14}$$

The ratio $m^{(21)}(v)$ for fixed Z increases from the value in (4.13) to a maximum, and then decreases monotonically to the values in (4.14). The maximum value of v is determined by the root of the equation ($(\gamma v)^{1/2}_{\max} = 0.791068980$)

$$5(1 + \gamma v^2) \operatorname{arc\,tg}(\sqrt{\gamma} v) = \sqrt{\gamma} v (5 + 3\gamma v^2)\,. \tag{4.15}$$

The values of v and $m^{(21)}$ at the maximum point are as follows:

$\gamma =$	1.1	1.2	4/3	1.4
$v_{max} =$	0.75425467	0.72214387	0.685085	0.66857531
$m^{(21)}_{max} / \sqrt{Z} =$	5.4894202	2.8050695	1.7279622	1.4576401
$\gamma =$	1.5	5/3	2	3
$v_{max} =$	0.64590511	0.61275939	0.55937024	0.45672388
$m^{(21)}_{max} / \sqrt{Z} =$	1.1863999	0.91354880	0.63743485	0.35271881

For $v \ll 1$ the ratio $m^{(21)} > 1$. In the region of moderately large v, and also in the region where $v \gg 1$, if

$$Z \gg v \geqslant (6Z)^{1/3} [\gamma (\gamma - 1)^2]^{-1/3}, \tag{4.16}$$

the ratio $m^{(21)} < 1$ and the induced radiation waves predominate; they are attenuated more slowly, and are propagated with greater velocity.

Formula (4.12) for $r^{(21)}$ is valid for any v in the range considered. The velocity and length of the waves induced by radiation are greater than the velocity and length of the acoustic waves and of pressure waves of the same frequency. The velocity of the waves is proportional, and their length inversely proportional, to $\sigma^{1/2}$. The ratio of velocities increases with increasing v and γ, and decreases with increasing Z. The ratios are $r^{(21)}$ and $r > 1$.

The ratio $\alpha^{(21)} \approx Z \gg 1$ throughout the entire range of v. Depending on v, it varies [1] as the function K'(v), and has a maximum

$$v_{max} = 1.514994 / \sqrt{\gamma}, \quad \alpha^{(21)}_{max} = 0.459756 \sqrt{\gamma} Z / (\gamma - 1). \tag{4.17}$$

4) In the region v = 0(z) the root of the characteristic equation is a small quantity. In first approximation Eq. (2.2) is valid, and the waves propagate according to the laws for a piezotropic medium. If the solution for a piezotropic medium is assumed as a first approximation, then the second and third approximations for any value of γ are determined by the expressions

$$\varepsilon_1 = \frac{(\gamma - 1)\zeta_1 [AD - BC + i(AC + BD)]}{C^2 - D^2},$$

$$\varepsilon_2 = \frac{(\gamma - 1) m_0^2 i\zeta_1 (2\gamma\varepsilon_1 - m_0^2) E - \gamma^2 \varepsilon_1^2 [m_0^4 v^4 + i\zeta_1 E^2]}{E (m_0^2 v^2 - i\zeta_1 E)},$$

$$A = m_{r0}^2 - m_{i0}^2 - v^2 (m_{r0}^4 - 6m_{r0}^2 m_{i0}^2 + m_{i0}^4),$$

$$B = 2m_{r0} m_{i0} [1 - 2v^2 (m_{r0}^2 - m_{i0}^2)],$$

$$C = (m_{r0}^2 - m_{i0}^2) v^2 - 2\zeta_1 m_{r0} m_{i0}, \qquad E = 1 - m_0^2 v^2,$$

$$D = 2v^2 m_{r0} m_{i0} + \zeta_1 v^2 (m_{r0}^2 - m_{i0}^2) - \zeta_1. \tag{4.18}$$

It is seen from (1.5) that for large values of Z in the vicinity of ζ_1^* the value of α_{10} is small, while the value of r is large. The waves are attenuated weakly on the acoustic wavelength, and are propagated at very great supersonic velocity. With increasing ζ_1, the value of α_{10} decreases, and α_1 and r increase. In this region the curves of $\alpha_{10}(v)$ (and also $\alpha_{10}(\zeta_1)$) for various γ again intersect, and after intersection, the curve for the larger γ will go higher. But the difference between $\alpha_{10}(v)$ for various values of γ is only a small quantity of order v^{-1}. The coefficient α_T in the region $\zeta_1 = 0(1)$ reaches a maximum, after which it decreases monotonically to α_T^* for $\zeta_1 \to \zeta_1^*$.

The ratios $m^{(21)} < 1$, and $r^{(21)} > 1$ for $\zeta_1 \to \zeta_1^*$. The ratio $m^{(21)}$ becomes less than 1 either for $\zeta_1 \ll 1$ if (4.16) is satisfied, or for $\zeta_1 = 0(1)$ in the opposite case; starting from a certain ζ_1 (for moderate or large v) the waves induced by radiation become predominant, and, with increasing v, this predominance increases ($m^{(21)}$ decreases and $r^{(21)}$ increases).

The ratio $\alpha^{(21)}$, with increasing v in the region of moderate values of ξ_1, reaches a second minimum, much greater than unity, after which it increases without limit for $\zeta_1 \to \zeta_1^*$.

§5. PROPERTIES OF WAVES INDUCED BY RADIATION

This section presents results from the solution of the characteristic equation for the entire range of values of γ, v, and Z.

Absorption Coefficient on the Acoustic Wavelength (Fig. 5). With variation of ζ_1 from 0 to ζ_1^*, for fixed γ and Z, the absolute values of m_r and m_i decrease from ∞ to m_r^* and 0. At small Z the curves of $\alpha_{10}(\zeta_1)$ and $\alpha_{10}(v)$ for the same Z but different γ intersect once each in the region $\zeta_1 = 0(1)$; the curve corresponding to the lower γ goes higher after intersection. The smaller the value of γ, the farther the curve goes to the right, and the lower the limiting value of α_{10}^*. With increasing Z, a second intersection occurs. For $Z \gg 1$ it occurs in the region Zv = 0(1). Up to the first intersection point, and after the second point, α_{10} is larger for larger γ and between the two, α_{10} is larger for smaller γ and the same Z and v. With increasing ζ_1 and Z, the dependence of the parameters on γ is smoothed. The larger the value of Z, the greater the range of ζ_1 or v in which the parameters coincide with their values at $\gamma = 1$. For each γ the curves of $\alpha_{10}(\zeta_1)$ are lower and go farther to the right, the larger the value of Z. The curves of $\alpha_{10}(v)$ have the same dependence on Z and γ, and their dependence on v is analogous to that on ζ_1. The solid lines in Fig. 5 show the dependence of $\alpha_{10}(\zeta_1)$ for $\gamma = 5/3$, the dashed lines show the position for $\gamma = 1$ and the dot-dash lines give the limiting values of α_{10}^*.

The velocity and wavelength with variation of ζ_1 from 0 to ζ_1^* (or of v from 0 to v^*) increase monotonically from 0 to ∞ (Fig. 6 for $\gamma = 5/3$). For sufficiently small ζ_1 (or v) the waves are subsonic, and their wavelength is shorter than that of acoustic waves, while beginning at certain values of ζ_1 (or of v), depending on γ and Z, they become supersonic and longer than the acoustic waves. The curves of $r(\zeta_1)$ and r(v) emerge from the coordinate origin with vertical tangents. The curve $z(\zeta_1)$ near the origin is the higher, the larger the value of Z and the smaller the value of γ, while close to ζ_1^* the curve is higher, and it (asymptotically) approaches infinity earlier, the smaller the value of Z and the greater the value of γ. Curves for the same γ, but different Z, or for the same Z, but different γ, intersect each other once (with the exception of the coordinate origin).

Optical Wave Number. The curves of $\eta(\zeta_1)$ emerge from the coordinate origin with a slope independent of Z and proportional to $(\gamma)^{1/2}$, they reach a maximum, and then monotonically approach the x-axis, as $\zeta_1 \to \zeta_1^*$; the larger the value of Z, and the smaller the value of γ, the closer the descending branch approaches from the left to the curve for a piezotropic gas.

The absorption coefficient α_T on the radiation free path, with increasing ζ_1 from 0 to ζ_1^* (of v from 0 to v^*), first increases from 0 to a maximum, in the region of moderate values of ζ_1, whose maximum depends on Z and γ (Fig. 7), and then decreases to α_T^* for any values of γ and Z. All the curves $\alpha_T(\zeta_1)$, $\alpha_T(v)$ are tangent to the y-axis at the origin of the coordinates. The curves $\alpha_T(\zeta_1)$ near the origin do not depend explicitly on Z and progress the more steeply, the larger the value of γ. The curve $\alpha_T(v)$ progresses the more steeply, the smaller the value of Z. Near ζ_1^* both curves decrease more rapidly for larger γ; the curve $\alpha_T(\zeta_1)$ decreases more rapidly for larger Z, and the curve $\alpha_T(v)$ decreases more rapidly for smaller Z. The maxima of the curves are located farther to the right, the larger the value of Z and the smaller the value of γ. Figure 7 shows the curves of $\alpha_T(\zeta_1)$ for $\gamma = 5/3$ and various Z. The dot-dash line (below the line Z = 10) corresponds to $\gamma = 1$ and does not depend on Z. The line for $Z \ll 1$, $\gamma = {}^5/_3$ breaks away when $\zeta_1 = 0.78561$, for $Z = 2.5$, $\gamma = {}^5/_3$ when $\zeta_1 = 1.2412$ for $\gamma = 1$ at $\zeta_1 = 1.3093$.

The true absorption coefficient α_1 for small v, when ζ_1 and Zv are simultaneously small, is close to 1, and is determined by (3.6). In the vicinity of v = 0, the quantity $\alpha_1 > 1$, if $Z \leq Z_0$, and $\alpha_1 < 1$, if $Z > Z_0$ (Z_0 = 8.08332, 6.23538, 5.36656, 5.14393, 4.92950, 4.8, 4.74342, 4.64758, for γ = 1.1, 1.2, ${}^4/_3$, 1.4, 1.5, 1.6, ${}^5/_3$, 2, respectively). If $Z \leq Z_0$, then α_1 increases monotonically from 1 to ∞ with increasing ζ_1 (or v) from 0 to ζ_1^* (or v^*) (Fig. 8 for $\gamma = {}^5/_3$). The curves rise the more steeply, the smaller the value of Z. For $Z = Z_0$, the curve $\alpha_1(\zeta_1)$ at the point $\alpha_1 = 1$, $\zeta_1 = 0$, exhibits tangency of second order with the line $\alpha_1 = 1$. If $Z < Z_0$, $Z < (5.4)^{1/2}\gamma$, then the larger the value of γ, the steeper the rise of the curves; if $Z > Z_0 > (5.4)^{1/2}\gamma$, then the larger the value of γ, the smaller the angle at which the curves emerge.

For $Z > Z_0$ and any γ, with increasing ζ_1 (or v) from 0 to ζ_1^* (or v^*), the coefficient α_1 decreases from 1 to a minimum, and then increases without bound. The minimum value of α_1 for Z very close to Z_0, and for $Z \gg 1$ is determined, respectively, by the formulas

$$\zeta_{1\,\min} = \sqrt{-{}^1/_3 c_1 / c_3}\,,$$
$$\alpha_{1\,\min} = 1 - {}^4/_3 \sqrt{-{}^1/_3 c_1^3 / c_3}\,, \tag{5.1}$$

$$v_{\min} = 3 / Z\,,$$
$$\alpha_{1\,\min} = (1 + \sqrt{2-\gamma})/(1 + \sqrt{\gamma})\,. \tag{5.2}$$

With increase of Z from Z_0 to ∞, the value of $\alpha_{1\min}$ decreases from 1 to the value of (5.3); $\zeta_{1\min}$ increases from 0 to a maximum value, and then decreases monotonically. The minimum α_1 is the lower, the greater the value of γ. The curve departs downward more steeply from the point $\zeta_1 = 0_1$, $\alpha = 1$, the larger the value of Z and the smaller the value of γ (if $\gamma > 2$, $Z > (5.4)^{1/2}\gamma$, then the curve is the steeper, the larger the value of γ). For $Z > Z_0$ and $0 \leq v \leq v_{11}$, the coefficient $\alpha_1 \leq 1$. If Z is close to Z_0, then $v_{11} = (-c_1/c_3)^{1/2} Z$. With increasing Z, the value of v_{11} increases from 0 to $[5(\gamma - 1)/(3\gamma)]^{-1/2}$. The quantity α_1 is never a small quantity; the wave shape is not maintained.

Comparison with Pressure Waves. Absorption. As long as the value of Z is small, the ratio $m^{(21)} \gg 1$ decreases with increasing ζ_1 or v (Figs. 9, 10 for $\gamma = {}^5/_3$). The pressure waves predominate. With increasing Z, the general form of the function $m^{(21)}$ changes, becoming close to that described in §4 for $Z \gg 1$, with one minimum, greater than unity, and a maximum. For a certain Z equality $m^{21} = 1$ is attained. For a given γ this is attained (with increasing Z) first for a certain value of Z^{**}, equal to the limiting value of Z^*, and occurs only at the point $\zeta_1 = \zeta_1^{**} \to \zeta_1^*$ (or $v = v^{**} \to v^*$). With further increase in $Z > Z^{**} \cong 2(3)^{1/2}\gamma/(\gamma - 1)$ the inequality $m^{(21)} < 1$ is satisfied for an increasingly larger segment of v (or of ζ_1), with its right-hand end at the point $v \to v^*$ ($\zeta_1 \to \zeta_1^*$). For $Z > Z^{**}$ the induced wave predominates in the range $v^{**} \leq v < v^*$, the predominance being greater for larger γ, Z, and v. Figure 10 shows the ratio of the absorption coefficient for thermal radiation waves and pressure waves for $\gamma = {}^5/_3$ (solid lines), $\gamma = {}^7/_5$ (dot-dash lines), for the values of Z shown on the curves; the limiting values of the ratio are shown by the dashed lines.

Comparison With Pressure Waves. Velocity. The ratio $r^{(21)}$ increases monotonically from 0 to ∞ with increasing v (or ζ_1) (Fig. 11 for $\gamma = {}^5/_3$). For small v, the pressure waves predominate also as regards velocity of propagation. For values of the parameters satisfying the inequality $m^{(21)} < 1$, the inequality $r^{(21)} > 1$ also holds true.

Comparison With Average Theories. Induced waves were examined in [8] with averaging over the directions of the equation of reaction transfer. The characteristic equation took the form (the averaging coefficients were included in q^0 and ζ_1^0):

$$\frac{1}{1-q^2} = 1 + \gamma i \zeta_1^0 \frac{1+m^2}{\gamma + m^2}\,. \tag{5.3}$$

From accurate calculation of the distribution of radiation with direction, it follows that waves will exist only in a limited range of values of $u(\gamma, Z)$ since averaging over direction leads to the existence of waves for any values of γ, Z and v. In averaged theory the result is that with variation of ζ_1 from 0 to ∞, the coefficient α_T increases monotonically from 0 to g^{-1} (g is the averaging coefficient). It follows from exact theory that α_1 increases from 0 to α_T^* with increase of ζ_1 from 0 to ζ_1^*, and reaches a maximum in this region. It has been shown in averaged theory that $\alpha_1 < 1$ for $Z < Z_0$, $1 \leq \gamma < 2$ and $\zeta_1 > 0$. It follows from exact theory that for γ close to 1 and $Z < Z_0$, the coefficient α_T is less than 1 (by a small quantity) in some range of low values of v. For small ζ_1 the left side of (5.1) and (5.4) for $g = 1/(3)^{1/2}$ differ by the small terms $0(|q|^4)$, and both theories give identical results; the divergence increases with increase of ζ_1. However, in many respects, the two theories give a convergent general picture of motion, even in the general case.

REFERENCES

1. V. A. Prokof'ev, "Attenuation of forced weak plane pressure waves in a gas with radiative heat transfer," PMTF [Journal of Applied Mechanics and Technical Physics], no. 3, pp. 8-16, 1966.
2. V. A. Prokof'ev, "The effect of radiation on the propagation of small disturbances in a viscous, heat-conducting fluid (hydrodynamic theory)," Izv. AN SSSR, OTN, no. 7, pp. 94-102, 1957.
3. V. A. Prokof'ev, "Absorption and dispersion of forced waves of very low and very high frequency with radiative heat transfer," Izv. AN SSSR, OTN, no. 12, pp. 15-23, 1958.
4. V. A. Prokof'ev, "Treatment of radiation in the hydrodynamic theory of the plane forced wave propagation of infinitely small amplitude," Vestn. Moskv. un-ta, Ser. matem. mekhan., astron., fiziki, khimii, no. 6, pp. 7-16, 1957.
5. V. A. Prokof'ev, "Infinitely small forced waves in a radiating barotropic medium," collection: Problems in Mechanics (ed. L. N. Sretenskii), Izd. MGU, no. 193, 93-130, 1961.
6. V. A. Prokof'ev, "The velocity of weak waves in a radiating gas," PMTF, no. 3, pp. 11-19, 1963.
7. E. Jahnke and F. Emde, Tables of Functions with Formulas and Graphs [Russian translation], Gostekhizdat, p. 47, 1948.
8. V. A. Prokof'ev, "Theory of propagation of thermal radiation waves of small amplitude, based on Euler's gasdynamic equations and allowing for radiative heat exchange," Vestn. Moskv. un-ta, no. 5, 55-66, 1964.

29 November 1966 Moscow